Is there still room for new developments in geostatistics?

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Georges Matheron lecture, IAMG

34th IGC, Brisbane, 8 August 2012

Matheron: books and monographs

- 1962-1963: *Treatise of applied geostatistics* (in French), Technip and BRGM editions, Paris
- 1965: *Regionalized variables and their estimation* (in French), Masson, Paris
- 1967: *Elements for a theory of porous media* (in French), Masson, Paris
- 1968: *Treatise of applied geostatistics* (in Russian), MIR, Moscow
- 1969: *Theory of random sets* (in French), Ecole des Mines de Paris
- 1969: *Geostatistics course* (in French), Mines Paris
- 1969: *Universal kriging* (in French), Mines Paris
- 1970: *Mathematical morphology* (in French), Mines Paris
- 1970: *The theory of regionalized variables and its applications*, Mines Paris
- 1972-1975: *Random sets and integral geometry*, Wiley, New York
- 1978-1989: *Estimating and choosing*, Springer, Berlin

Three recent developments

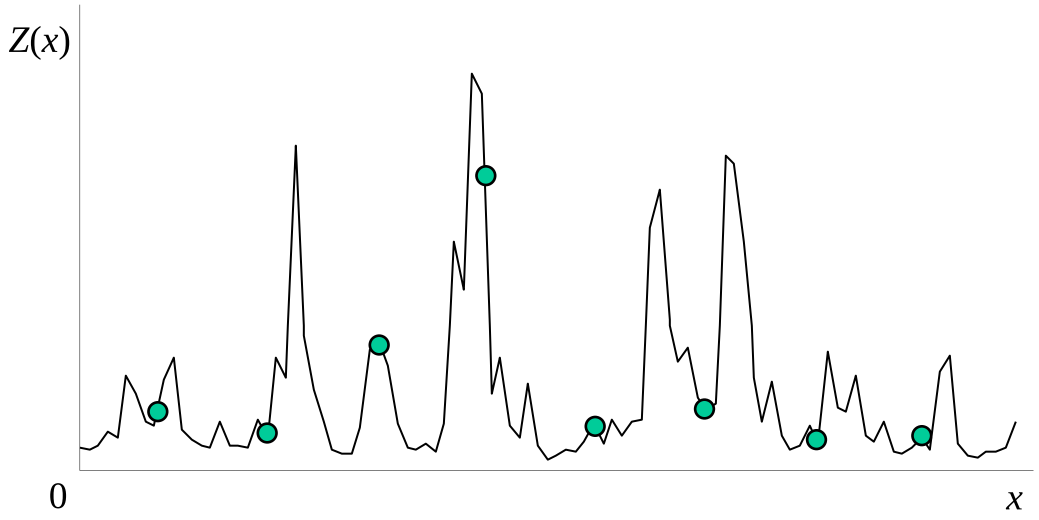
- Dealing with outliers
- Modeling a change of support with the discrete Gaussian model
- Simulating a Gaussian random vector

Dealing with outliers

J. Rivoirard, X. Freulon, et al.

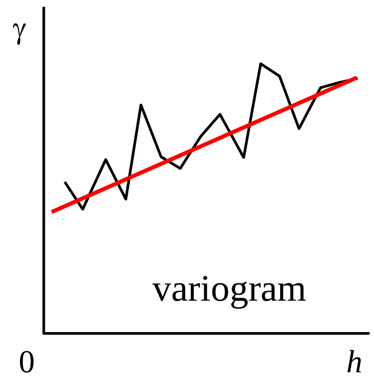
Kriging in the presence of outliers

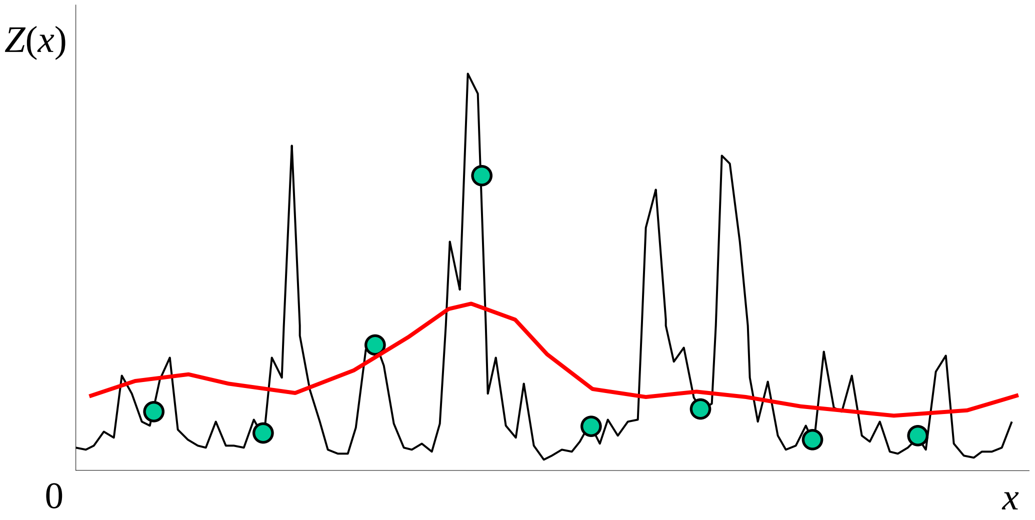
- Some variables (gold grade, concentration in a pollutant) have a histogram with a long tail. The data include some high values or outliers.
- How can we interpolate?



Standard approach

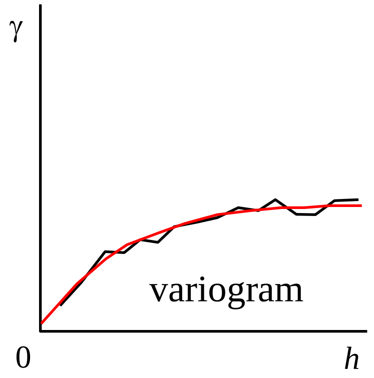
- Lack of robustness of the sample variogram
- Large nugget effect
- Large kriging variance

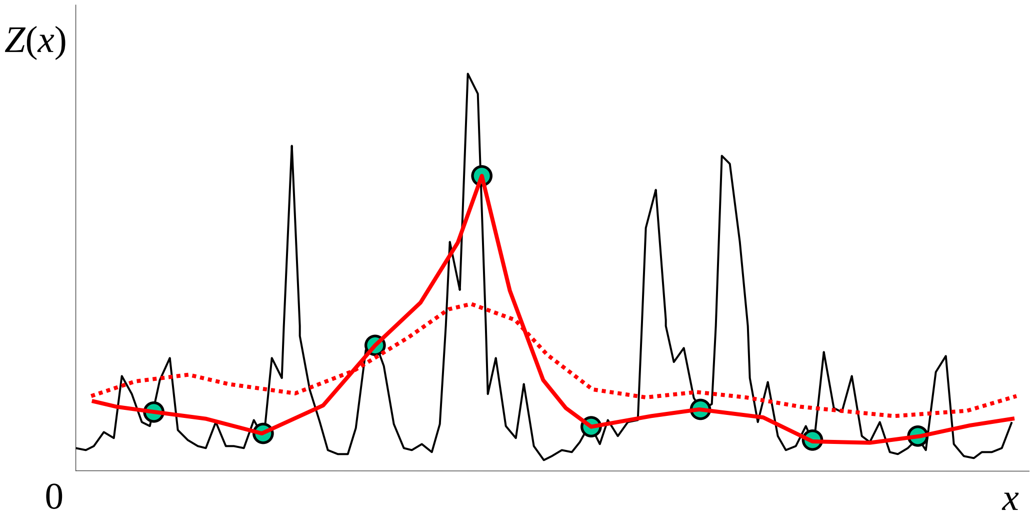




Ignoring the outlier in the variogram calculation
Kriging with all data

- Robust, but biased, variogram
- Inconsistency between variogram and kriging
- Extends the influence of the outlier data on the basis of the structure of the low grades

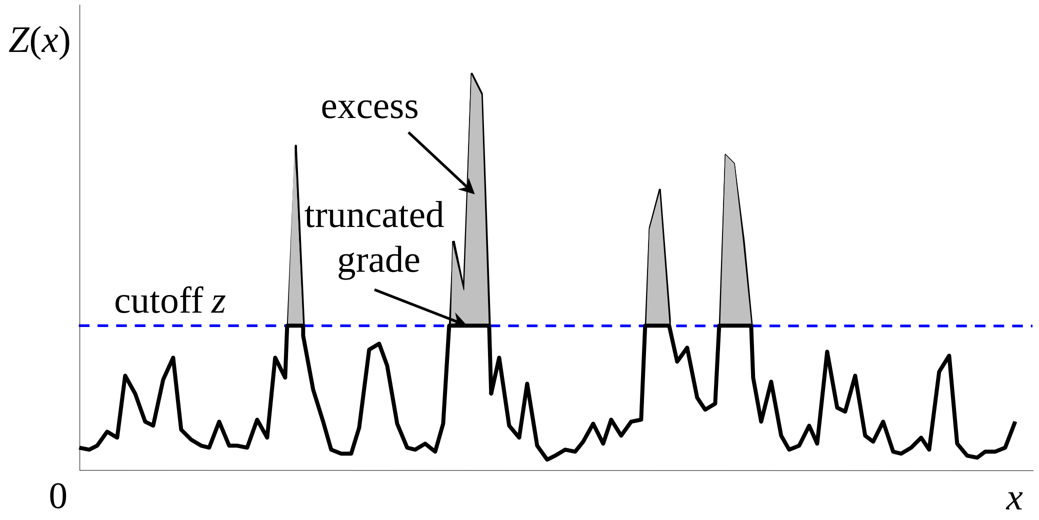




Introduction of a cutoff

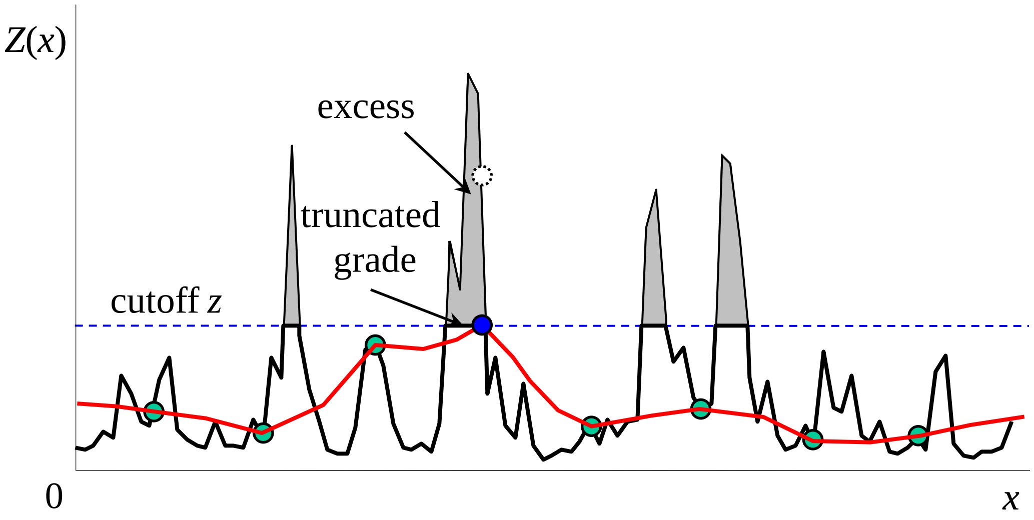
A cutoff z separates $Z(x)$ into

- Truncated grade $\min(Z(x), z)$
- Excess



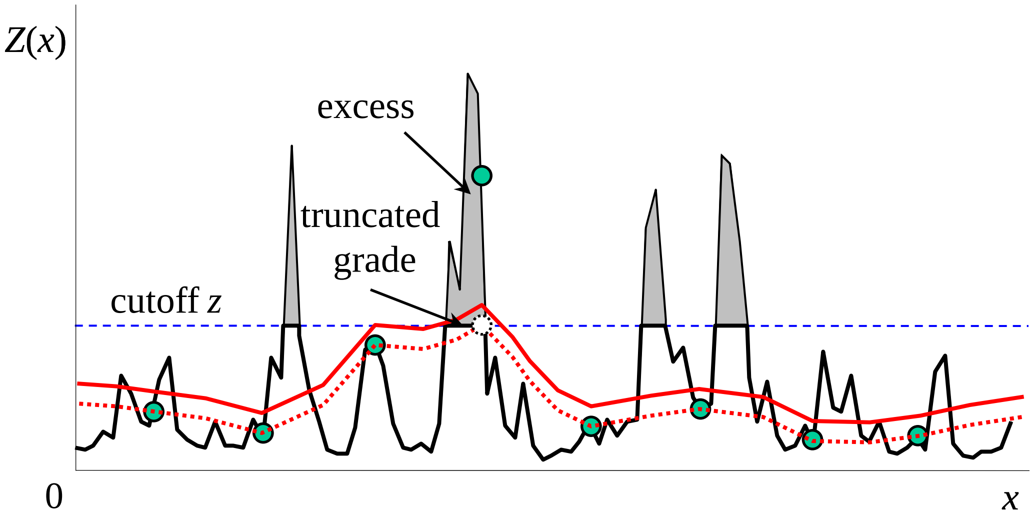
Truncation of large values

- Application of the standard approach to the truncated grade
- More interpretable variogram
- Annihilates the excess



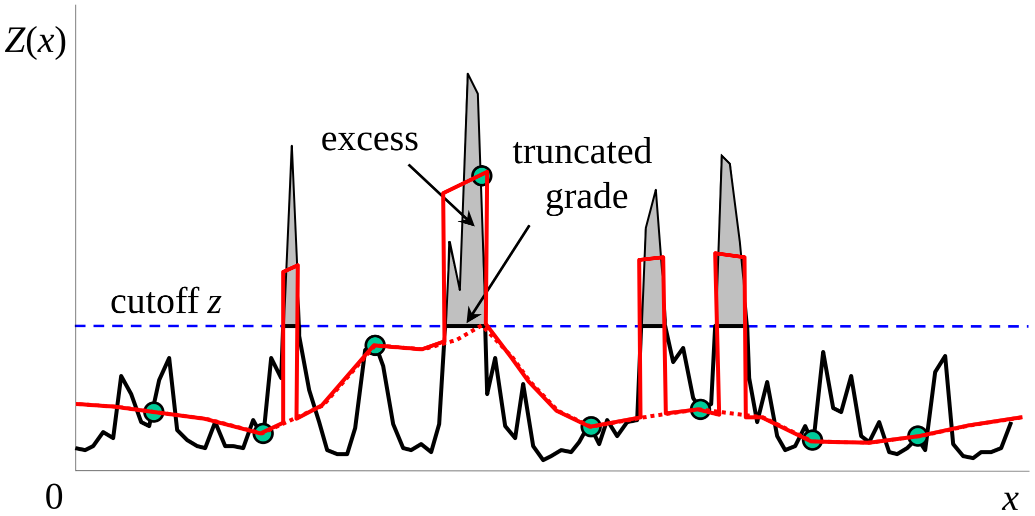
Global spreading of the excess

- Excess considered as a nugget effect and spread over the whole domain
- Consistent globally
- Spreads the excess even in areas where there is no excess



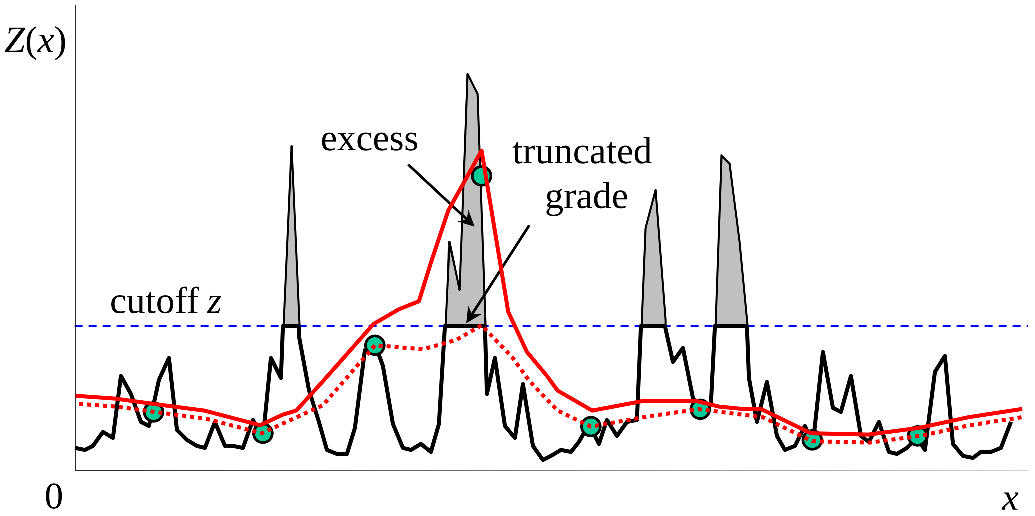
Spreading the excess where there is excess

- OK but requires knowing where there is excess



Spreading the excess where excess is likely

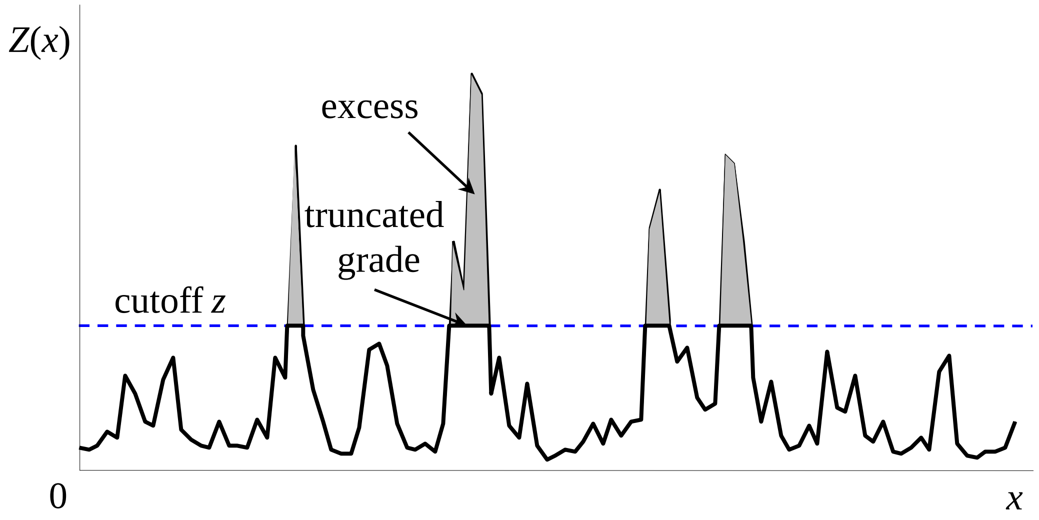
- Estimate the indicator of excess
- Spread the excess proportionally to the indicator estimate



Validity of the approach

$Z(x) = T(x) + m_E\, I(x) + R(x)$

- $T(x) = \min(Z(x), z)$ (truncated grade)
- $I(x) = 1_{Z(x)>z}$ (indicator of the excess)
- $R(x)$: zero-mean residual
- m_E : conditional mean of the excess



Validity of the approach

The model

$$Z(x) = T(x) + m_E\, I(x) + R(x)$$

is specially interesting when:

- R is spatially uncorrelated with T and I (no edge effect in the high-value zone)
- R is not structured

Indeed the final estimator is then

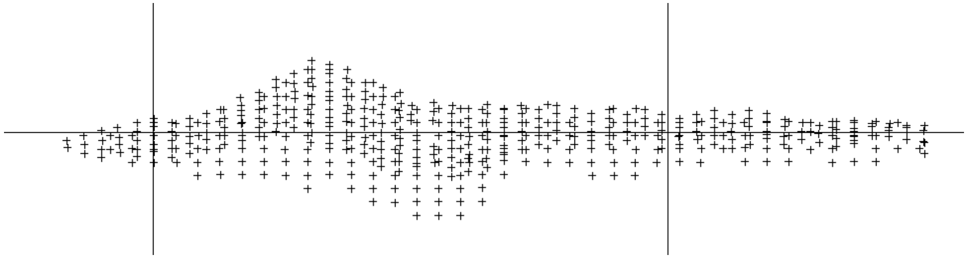
$$Z^*(x) = T^*(x) + m_E\, I^*(x)$$

(T^* and I^* obtained by cokriging)

It is free from high grades.

Application

- Gold deposit (vertical vein)
- Very skew distribution:
  mean = 1.76 g/t, σ/m = 7.74, maximum = 443 g/t

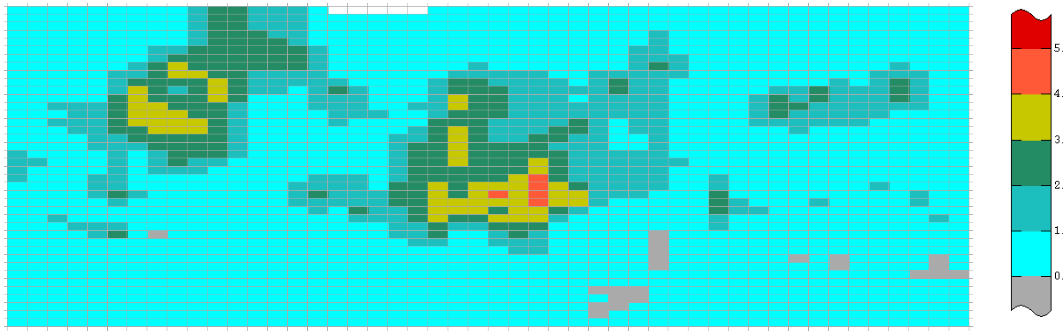


Top view of the deposit:
Trace of the cross-section and location of the blast holes

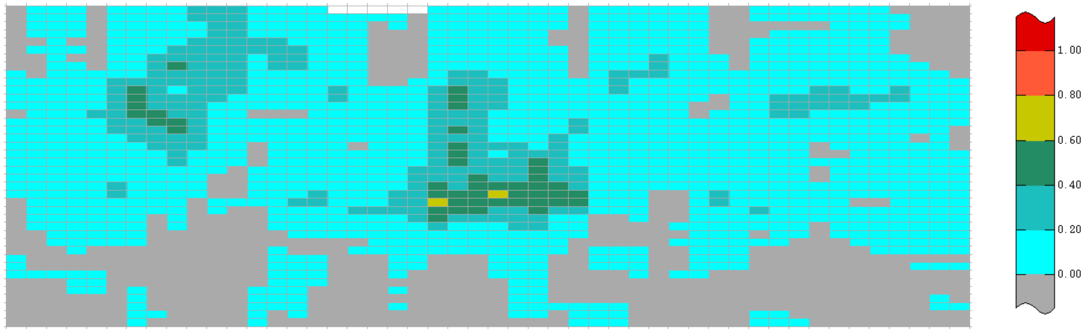
Application

- Cokriging of indicator of excess and truncated grade (cutoff: 5 g/t)

Truncated grade



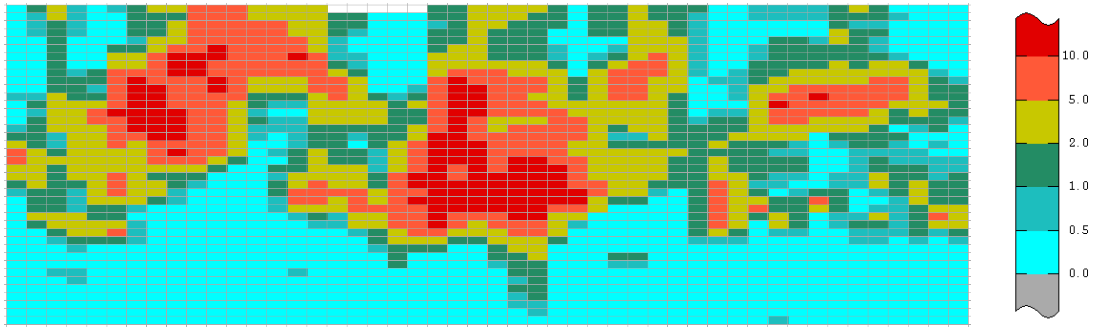
Indicator of excess



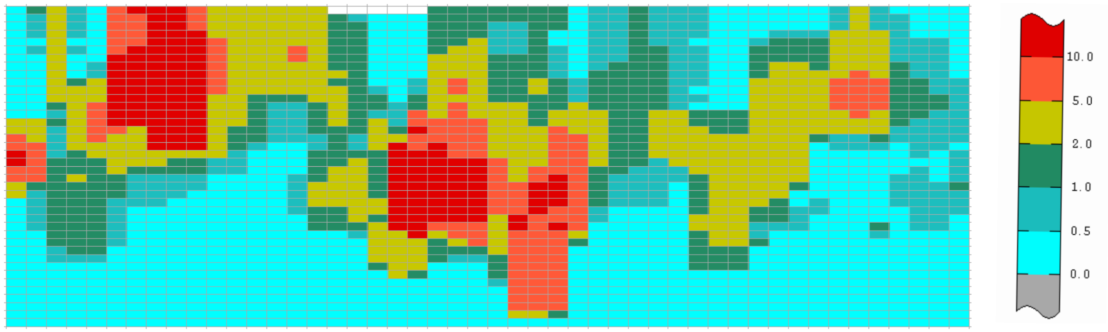
Application

- Final cokriging estimates compared with direct kriging

Final cokriging estimates

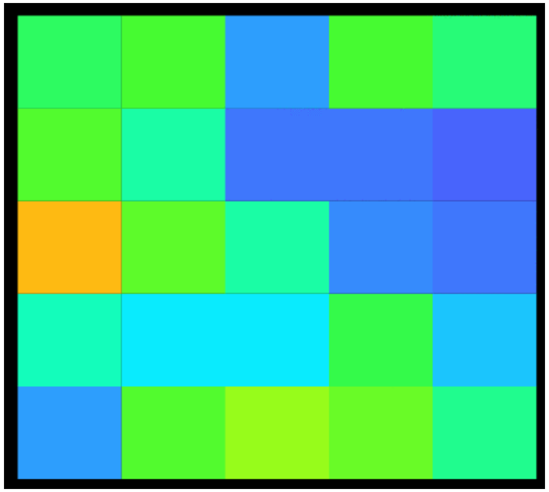


Direct kriging estimates

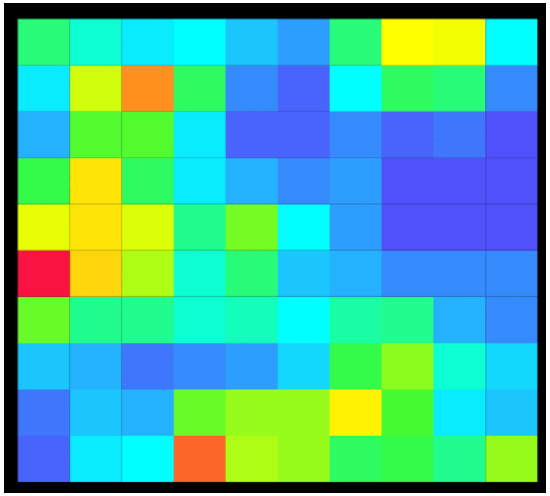


Predicting a change of support with the discrete Gaussian model

Support and selectivity



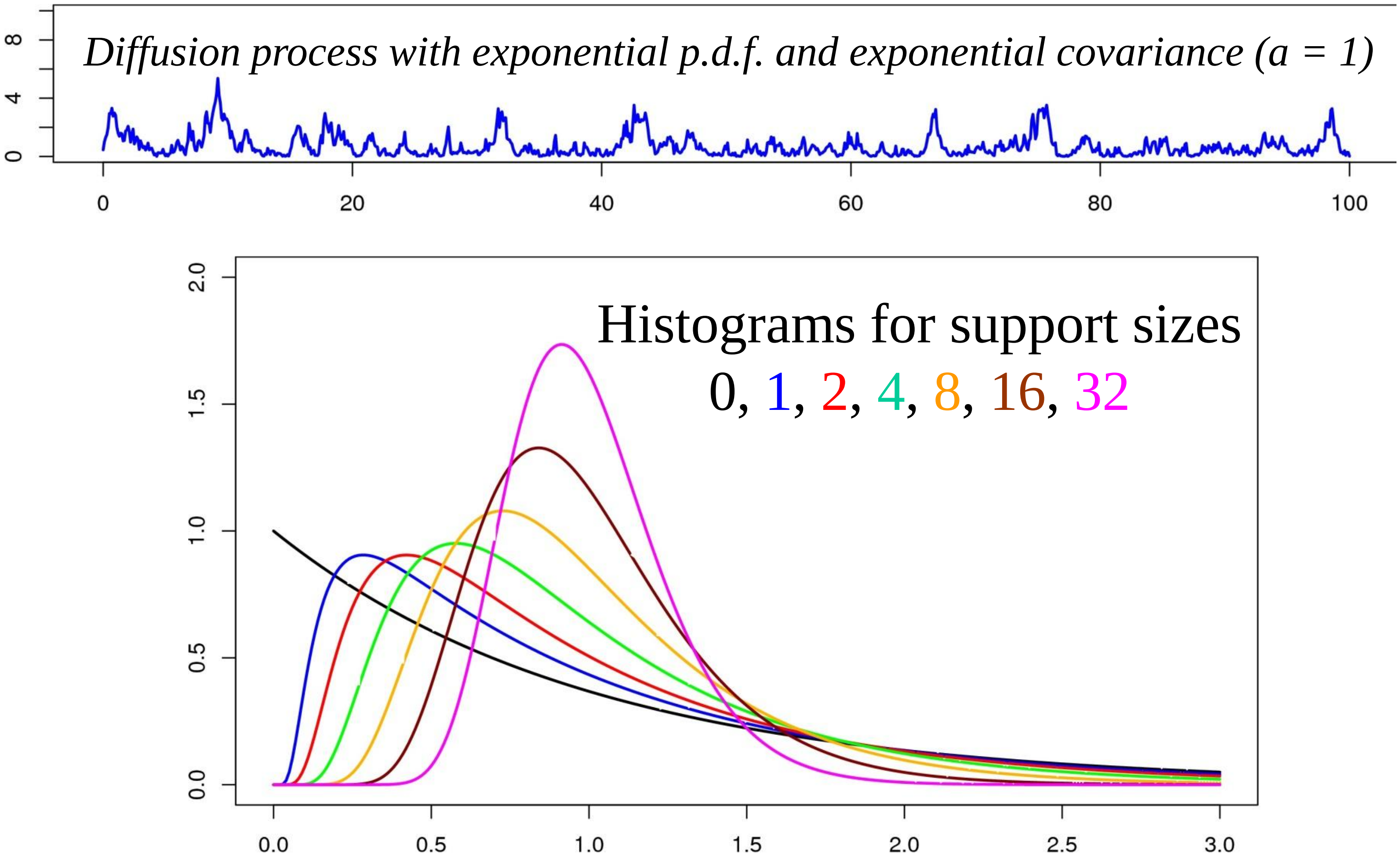
low-constrast variations



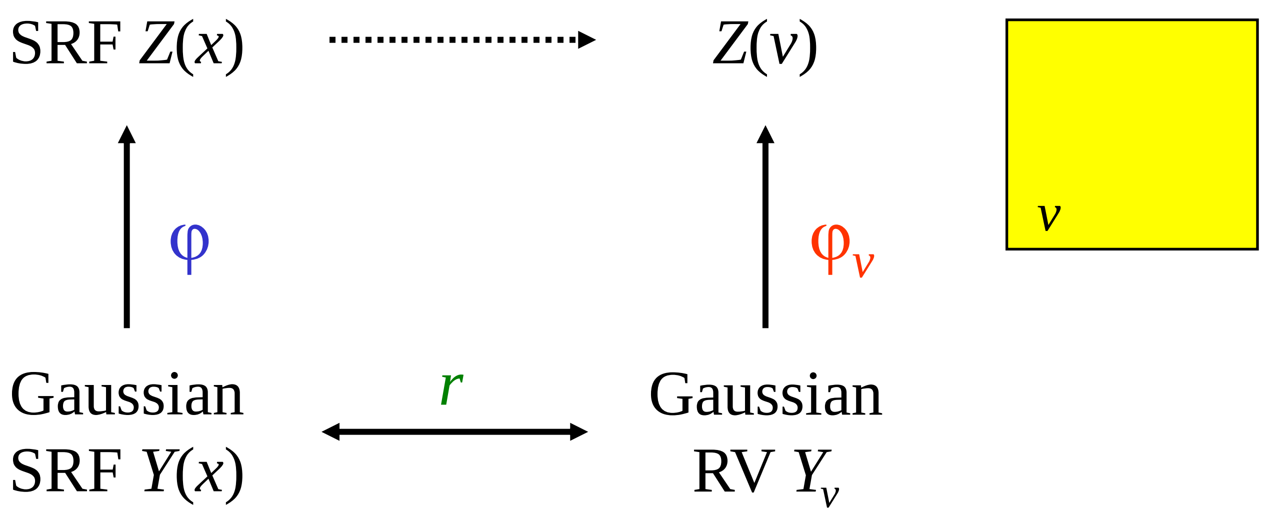
high-contrast variations

Selectivity depends on block size

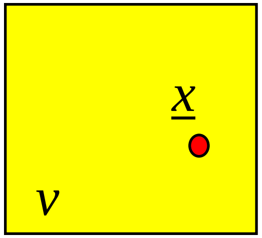
Support and grade distribution



Principle of the discrete Gaussian model



Assumptions of the discrete Gaussian model



Model DGM1 (Matheron, 1976)
Assumption: The pair ($Y(\underline{x})$, Y_v) is bi-Gaussian
($\underline{x}$: random point in v)

Characterized by the correlation coefficient r of $Y(\underline{x})$ and Y_v

Model DGM2 (Emery, 2007)
Additional assumption: The pair ($Y(\underline{x})$, $Y(\underline{x}')$) is bi-Gaussian
($\underline{x}$, $\underline{x}'$: independently random in v)

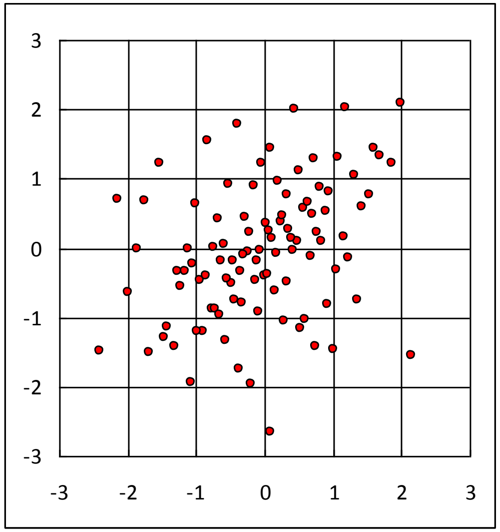
Offers the facility that $Y_v = Y(v) / r$, where r is the correlation coefficient of $Y(\underline{x})$ and $Y(v)$

These assumptions are approximations

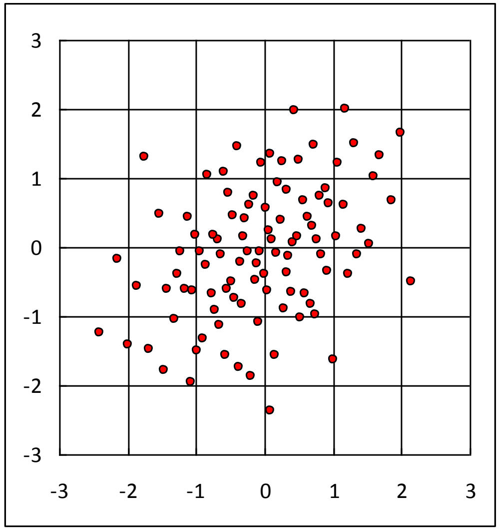
Check of the additional assumption of DGM2

(1D, triangle covariance, segment length = range)

Sample of the true
($Y(\underline{x})$, $Y(\underline{x}')$) distribution



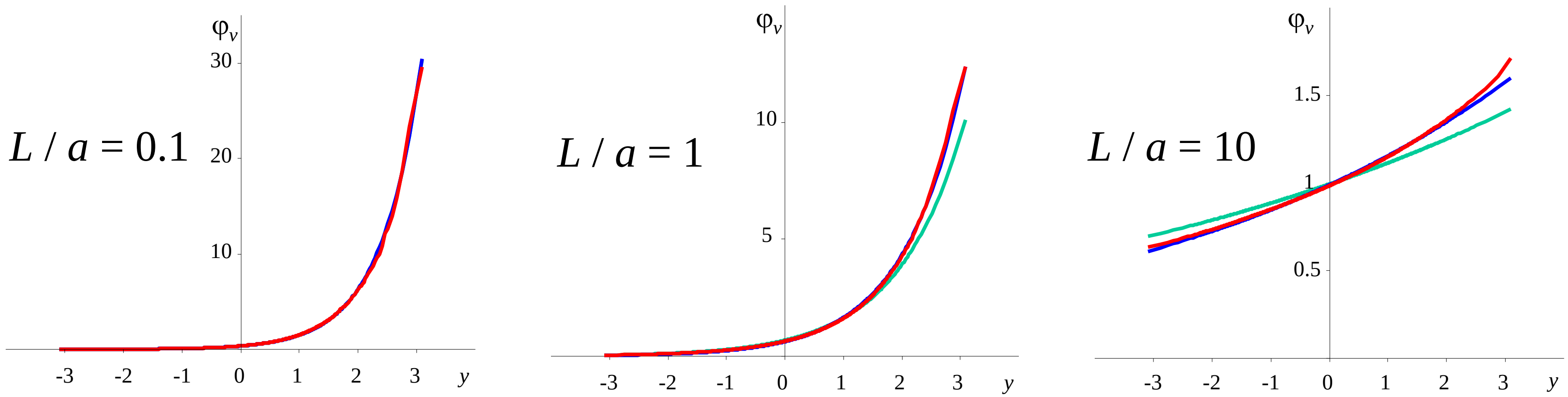
Sample of the approximate
($Y(\underline{x})$, $Y(\underline{x}')$) distribution



Some dissimilarity

Validity of DGM models: Case of a lognormal SRF

- DGM model = permanence of lognormality
- Example of a logarithmic standard deviation $\sigma = 1.5$
- 2D, square $L \times L$, range a



Comparison of "true" φ_v with φ_v given by DGM1 and DGM2

True φ_v
DGM1
DGM2

Conclusions

DGM1 is more robust than DGM2:

- DGM1 gives a good answer up to a large logarithmic variance.
- DGM2 can be used safely for a small logarithmic variance, and otherwise for a block of small size with respect to the range.

DGM2 facilitates calculations in case of:

- multiple supports
- polymetallic deposit
- information effect

Simulating a Gaussian random vector

C. Lantuéjoul and N. Desassis

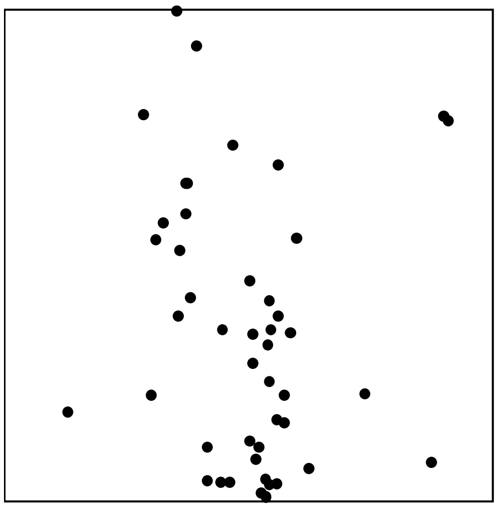
Initial motivation

- Secondary diamond deposits
- Simulation of the number of diamonds in blocks
- Data measured in blocks with various supports

Cox process

- Poisson point process with random intensity (or potential) $Z(x)$
- $Z(x)$ = transform of a Gaussian SRF $Y(x)$
- $Z(v_1)$, $Z(v_2)$, … obtained through DGM2

- Conditional simulation: requires the simulation of a large-size Gaussian vector



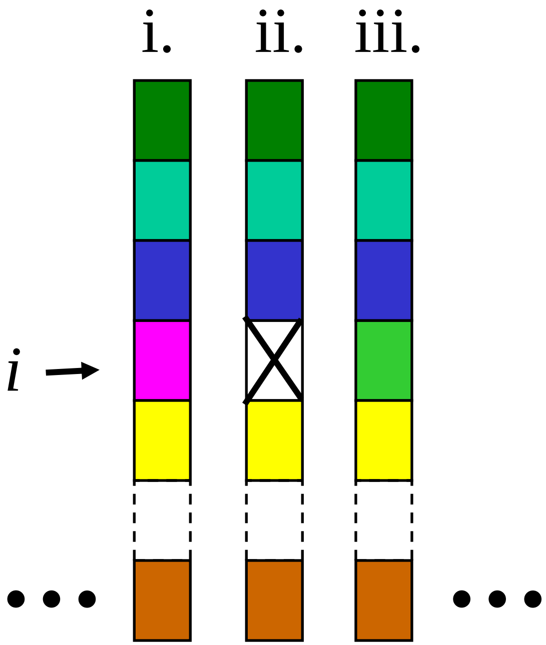
Simulating a large-size Gaussian vector with a given covariance matrix

- Objective:
  Simulate a Gaussian vector $\mathbf{Z} = (Z_1, Z_2, \ldots, Z_N)$ with zero-mean unit-variance components and correlation matrix $\boldsymbol{\rho} = [\rho_{ij}]$

Direct approach: Cholesky decomposition

- Well-known solution:
  - Decompose ρ into the product **A A**' where **A** is a lower triangular matrix
  - Select a Gaussian vector **U** with independent standard normal components
  - Take **Z** = **A U**
- Limited to a reasonable *N*

Iterative approach: Gibbs sampler



After initialization of vector **z** (e.g., **z** = **0**):

i. Select a component, say *i*

ii. Delete the value of this component

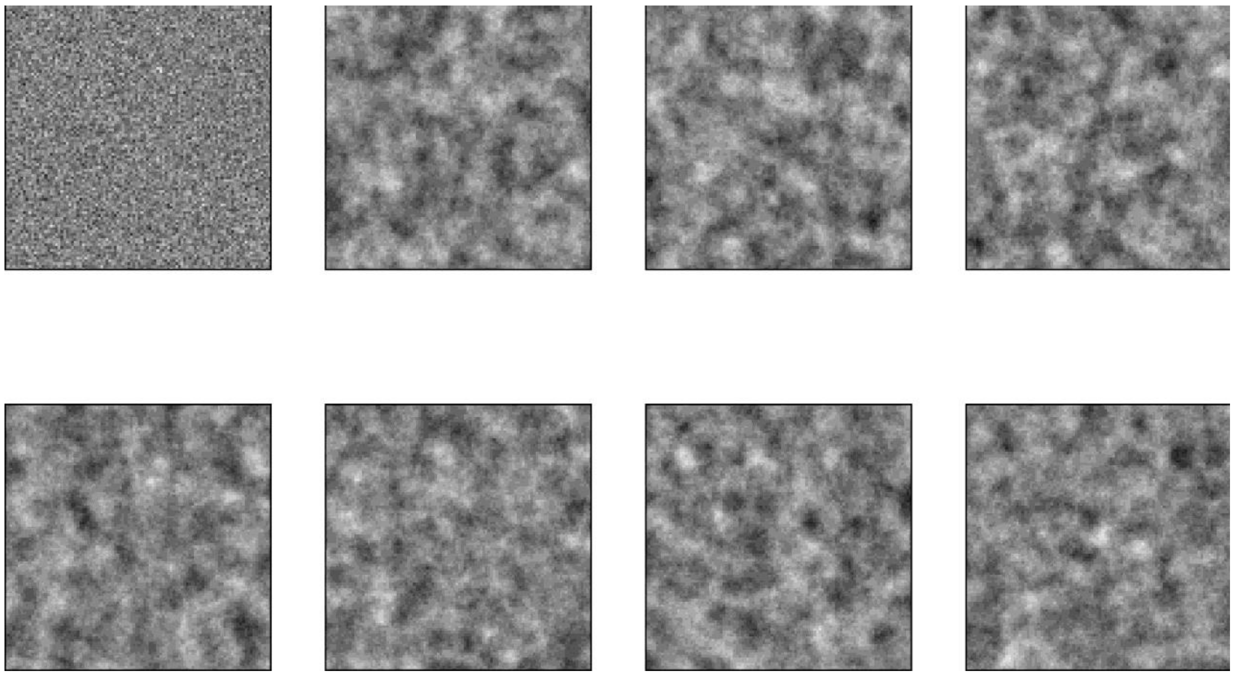
iii. Choose a new value from the conditional distribution of Z_i given the other components

iv. Go to i.

- The parameters of the conditional distributions derive from the inverse ρ^{-1}
- Can diverge if one uses the conditional distribution from a subset of the data (moving neighborhood)

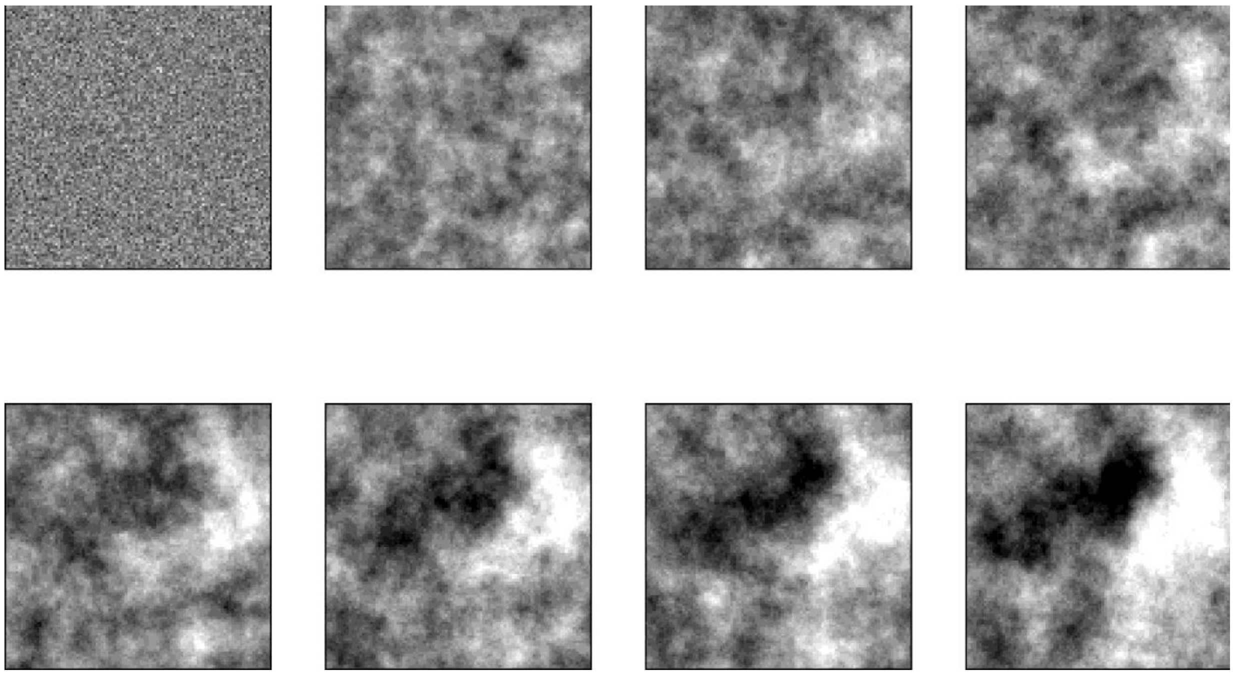
Iterative approach: Gibbs sampler

Grid 100×100, spherical variogram with range 10, neighborhood 15×15



Iterative approach: Gibbs sampler

Grid 100×100, spherical variogram with range 10, neighborhood 5×5



Reversing the viewpoint

- Two useful properties:
  - ✓ If ρ is a covariance matrix, its inverse ρ^{-1} is a covariance matrix.
  - ✓ If the vector **Y** has mean **0** and covariance ρ^{-1}, **Z** = ρ **Y** has mean **0** and covariance ρ.
- A solution:
  - ✓ Use the Gibbs sampler to simulate **Y**; then, derive **Z**.

- Requires the inverse $(\rho^{-1})^{-1}$, which is known (it is nothing but ρ)

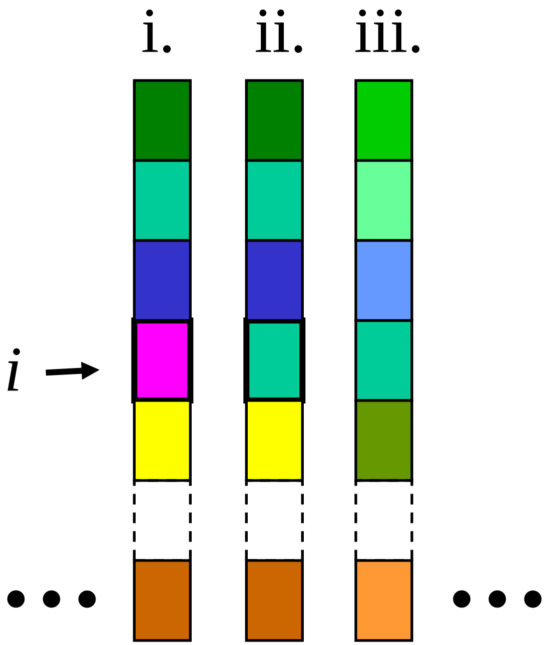
Suppressing the reference to **Y**

- Suppose that **Z** has mean **0** and correlation matrix ρ.
- If the component Z_i is changed into Z_i', possibly correlated with Z_i but conditionally independent of the other Z_j's, let us consider

$$Z_j' = Z_j + \rho_{ji}\,(Z_i' - Z_i) \qquad j \neq i$$

- It can be shown that **Z**' also has covariance matrix ρ.

Propagation algorithm



After initialization of vector **z** (e.g., **z** = **0**):

i. Select a component, say *i*

ii. Choose a new value for this component

iii. Propagate its influence on the other components

iv. Go to i.

- Different strategies for the choice of the new value
- Does not require the inverse ρ^{-1}

Propagation algorithm

- Achieves what seemed impossible:
  Simulate without inverting the covariance matrix
- Can therefore be used to simulate very large vectors

Conclusion

- It is worth revisiting standing problems
- What seems impossible may become straightforward once a sound solution has been found
- Is there still room for new developments in geostatistics?
  - Yes!
  - and even in classical geostatistics!

Special acknowledgements to
Georges Matheron (1930–2000)
who continues to be a source of inspiration

Georges Matheron lecture, IAMG

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